Progressive Education Society's
Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16
External Examination – Nov./ Dec. 2022
Faculty: Science and Technology

Program: B.Sc. Blended Semester: III **SET: A**
Program (Specific): **Course Type: Core**
Class: S.Y.B.Sc. **Max.Marks :50**
Name of the Course: MATHS 3 Vector calculus and Probability and Statistics I
Course Code: MTH 301

Instructions to the candidate:

1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1) Select correct option. **[10]**

i) A matrix having only one column is called _____

a) Scalar matrix b) Diagonal matrix c) Row matrix d) Column matrix

ii) If $u = (3,4)$ then $||u|| =$ _______.

a) 5 b) 3 c) 1 d) 4

iii) Two vectors u and v are said to be orthogonal if $< u, v > =$ _____.

a) 1 b) 2 c) 0 d) -1

iv) The eigen values of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are _____.

a) 1,1 b) 1,-1 c) 0,1 d) 0,-1

v) For the vector valued function $\overline{r}(t) = (t^2 - 3t)\overline{i} + (4t + 1)\overline{j}$, $\overline{r}(0) =$ _____.

a) 0 b) $\overline{j}$ c) $3\overline{i}$ d) $4\overline{j}$

vi) $\int_0^1 x\, dx =$ _____.

a) 0 b) 1 c) 2 d) $\frac{1}{2}$

vii) If $y = x^2$ then $\frac{dy}{dx} =$ ___.

a) $2x$ b) 2 c) x d) 0

viii) If $\overline{F}(\overline{r}) = 2x\overline{i} - 2y\overline{j}$ then $div\, \overline{F} =$ ______.

a) 1 b) -1 c) 0 d) 2

ix) A function $f(x)$ is said to be an even function if $f(-x) =$ ______.

a) 0 b) $-f(x)$ c) 1 d) $f(x)$

x) $\sin x$ is a periodic function of period $L =$ _____.

a) 2π b) π c) 0 d) -2π

Q.2) Answer the following (Any 10) **[20]**

i) State Cayley-Hamilton theorem

ii) Find determinant of the matrix $\begin{bmatrix} 4 & 3 \\ 0 & 2 \end{bmatrix}$.

iii) If $< u, v >=- 3$, $||u|| = \sqrt{19}$, $||v|| = 3$ then find the angle between u and v.

iv) When a set of vectors is said to be orthonormal?

v) Compute the curl of the vector field $\overline{F}(\overline{r}) = yz\overline{i} + xz\overline{j} + xy\overline{k}$.

vi) Define Vector Valued Function and give one example of it.

vii) Compute the partial derivative of $\overline{F}(\overline{r}) = x^2\overline{i} + y^2\overline{j}$ with respect to x and y.

viii) Find the gradient of $f(\overline{r}) = x^2 - y^2$.

ix) Define Polar Co-ordinates?

x) State Green's Theorem'.

xi) Show that $f(x, y) = x^2 - y^2$ is a harmonic function.

xii) Define Continuity of a vector valued function.

Q.3) Answer the following (Any 4) **[20]**

i) Find Characteristic Polynomial and all eigen values of the matrix

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 2 & 6 & 1 \end{bmatrix}.$$

ii) For the scalar field $f(\bar{r}) = y$ where $\bar{r} = x\bar{i} + y\bar{j}$ and the curve is $\bar{a}: [0, \pi] \to \boldsymbol{R}^3$, defined by

$\bar{a}(t) = cost\bar{i} + sint\bar{j}$, evaluate $\int f\, ds$.

iii) Find $\bar{f}(2)$ *where* $\bar{f}(t) = (t^2 + 1)\bar{i} + (4t - 3)\bar{j} + (2t^2 - \frac{1}{2}t)\bar{k}$ is continuous at t=2.

iv) Evaluate $\int (x^2 + e^x)dx$ along the path Γ=[0.4].

v) Compute the total derivative of $\bar{a}(t) = cost\bar{i} + sint\bar{j} + e^t\bar{k}$.

vi) Evaluate : $\lim_{t \to 0} [\,(1 + 3t)^{\frac{1}{t}}]i + \frac{sin3t}{t}\bar{j}$.